STREAM-FUNCTION USED FOR CURRENT-LINES' CONSTRUCTION IN 2-DIMENSIONAL DC MODELING

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The stream function described is employed for the presentation of 2D DC modeling results. The 2D model is understood as a 2D medium with linear current electrodes, oriented along the inhomogeneities' strike direction. In this case both the medium and the electric field depend on two space coordinates only. Modeling becomes much easier than considering point current electrodes, where the electrical field always is three-dimensional. Meanwhile the actual results of such modeling are qualitatively equivalent to 3D modeling with point electrodes, as long as the measurements are conducted across the objects.

The classical modeling presentation is in apparent resistivity which reflects an electric field distribution on the earth's surface. Quite often the connection of measured anomalies with a geoelectrical model is rather complex (fig. 1, A and C). The visualization of DC current lines simplifies understanding of the electric field's structure. Current lines are used in almost each textbook, but a practical technique for their construction is usually not included.

The evident way for drawing current-lines is the step by step continuation of a line from some point along the electric field direction. The practical realization of such approach is not trivial. For a 2D field it is possible to use the stream-function. This function is often used in EM field modeling [flux function, Berdichevsky, 1984]. A contour map of the stream-function corresponds to the stream-line distribution. Thus the problem of current streamlines' construction is reduced to the calculation of the stream-function in the research area. This can be achieved by calculating secondary surface charges, which are determined at 2D modeling, using Fredholm's integral equation of the second type relatively of electric field [Escola, 1979].

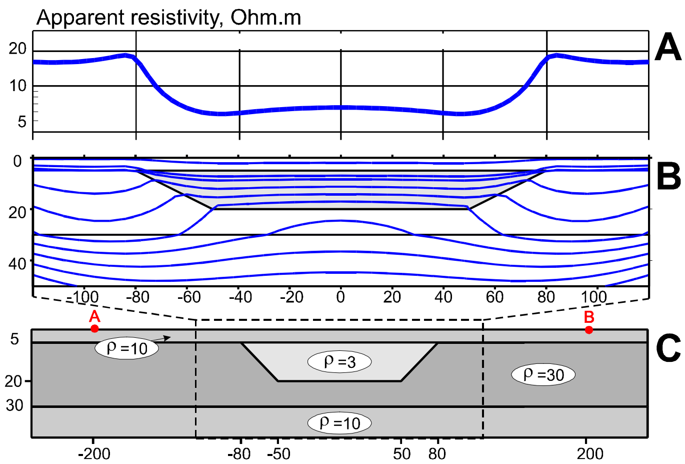


Fig.1. Apparent resistivity for a gradient array (A), DC current lines for central part of model (B) and the model with the current electrodes' position (C).

The stream-function's (ψ) physical definition is the difference between stream-function's values in two points in space, which is equal to the electric current intersecting a curve connecting them:

$$\psi(B) - \psi(A) = \int_A^B \mathbf{i}(l) \cdot \mathbf{n}(l)\, dl = \int_A^B \sigma(l) \cdot \mathbf{E}(l) \cdot \mathbf{n}(l)\, dl \ , \qquad (1)$$

where **i** is the vector of electric current density , **n** is the normal vector to the AB curve, **E** is the electric field intensity and $\boldsymbol{\sigma(l)}$ is the electric conductivity in a point *l*. The value of the integral (1) does not depend on the integration's path in regions without current sources. Therefore the stream-function can be calculated by integration from any point. Assuming that the stream-function is equal zero at point M, subsequent for any point K we can write an equation:

$$\psi(K) = \int_M^K \sigma(l) \cdot \mathbf{E}(l) \cdot \mathbf{n}(l)\, dl \ . \qquad (2)$$

The electrical field is the sum of a primary field of current from the current electrodes and a secondary field of charges from the modeled body's surface (surface charges). If the density of surface charges is known from 2D modeling, the integral (2) can be calculated analytically. The connection of stream-line distribution with apparent resistivity (ρ_a) depends on the kind of array. For pole-dipole array with ideal MN dipole the following relation is valid and exact:

$$\rho_a = \rho_{MN} \frac{j_{MN}}{j_0}, \qquad (3)$$

ρ_{MN} and j_{MN} are the resistivity and current density in MN center, j_0 is the current density for a uniform half-space. Therefore, if the upper layer is homogenous ($\rho_{MN} = \text{const}$), then ρ_a is proportional to the current density. This approach clarifies the ρ_a physical explanation as shown in fig.1A. The current distribution around conductive object (fig.1B) results in changes of current density on the surface and this finds it's expression in ρ_a anomalies. ρ_a for pole-pole array is related to secondary charge distribution. An expression can be derived for the potential value in point M (U_M): $U_M = U_0 + U_S$, (4)

where U_0 is the potential of the primary electric field, U_S is the potential of the secondary electric field, defined by the surface charge distribution. The surface charge density (Σ) on the boundary of the inhomogeneity is determined from the boundary condition (Strattom 1941, p.163):

$$\frac{\Sigma}{\varepsilon_0} = E_n^1 - E_n^2 = j_n(\sigma^1 - \sigma^2). \qquad (5)$$

Thus the surface charge density reveals a distribution of streamlines (fig.2). The upper left corner of the object is less expressed in the apparent resistivity than the upper right. Both corners will be expressed clearer in the case of a conductive object.

The best way to visualize surface charge distributions, is the ρ_a cross-section for a pole-pole array (2B), corresponding to measurements with a buried potential electrode. Maxima and minima of such cross-section show a maximum density of positive and negative surface charges.

In our practice stream-line function is used both for educational tasks and practical investigations. We can calculate on its base current lines, potential lines and apparent resistivity isolines in vertical cross-section. Like in formula 4 it is possible to separate potential, electric field and apparent resistivity into normal and anomalous parts and draw these lines distributions near anomalous object. Possibilities of this approach are listed in the table below and at some examples shown at fig.3-10.

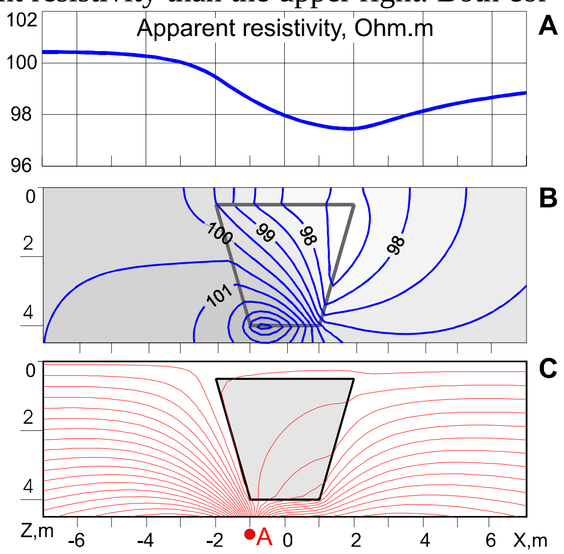
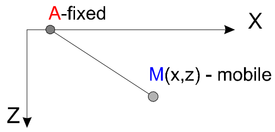
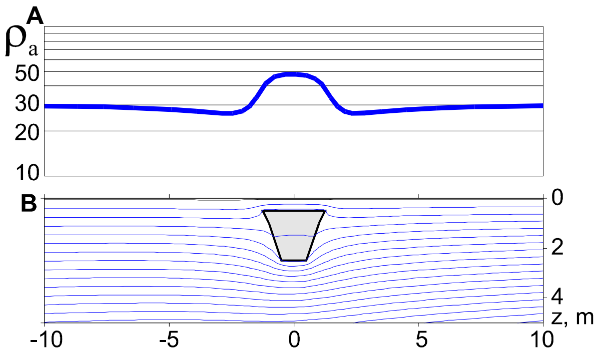


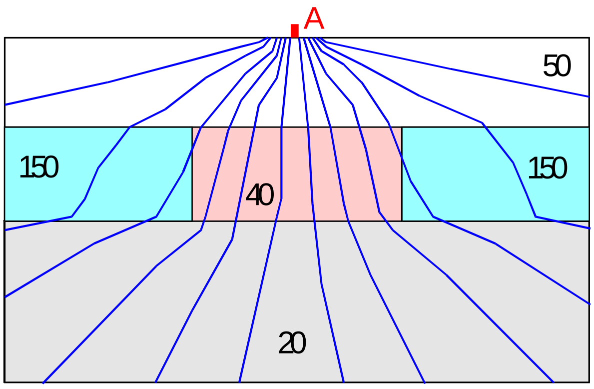
Fig. 2. The model with highly resistive object (ρ_{body}=1000 Ωm, ρ_{medium}=100 Ωm) for pole-pole array with buried current electrode. Apparent resistivity on the surface (A), in the cross-section (B) and DC stream-lines (C).

Table. DC_flow possibilities in calculation and presentation

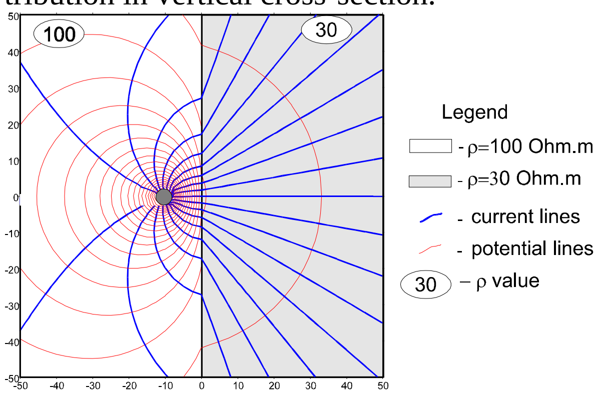
measured field's component	**apparent resistivity**	**field's presentation and examples' numbers**
Electrical field ΔU_{MN}	$\rho_a = K\frac{\Delta U_{MN}}{I} \xrightarrow[\lvert MN\rvert \to 0]{} \frac{j_{MN}\cdot \rho_{MN}}{j_{normal}}$	The stream-line distribution
Electrical potential U_M	$\rho_a = K\frac{U_M}{I} = \rho_0 + \rho_{anomalous}$ Primary field (for ρ_0); Field of secondary charges (for $\rho_{anomalous}$)	1. The stream-line distribution 2. $\rho_a(x, z)$ cross-section 
Vector of electrical field $\vec{E}_{observed} = \frac{\Delta U_x^{MN}}{\lvert MN\rvert}\vec{1}_x + \frac{\Delta U_y^{MN}}{\lvert MN\rvert}\vec{1}_y$	$\vec{\rho}_{observed} = \frac{\vec{E}_{observed}}{\lvert \vec{j}_{normal}\rvert}$	The stream-line distribution
	$\vec{\rho}_{anomalous} = \frac{\vec{E}_{observed} - \rho_a^{1D}\cdot \vec{j}_{normal}}{\lvert \vec{j}_{normal}\rvert}$	The stream-line distribution of anomalous electrical field $\vec{E}_{anomalous} = \vec{E}_{observed} - \vec{E}_{normal}$



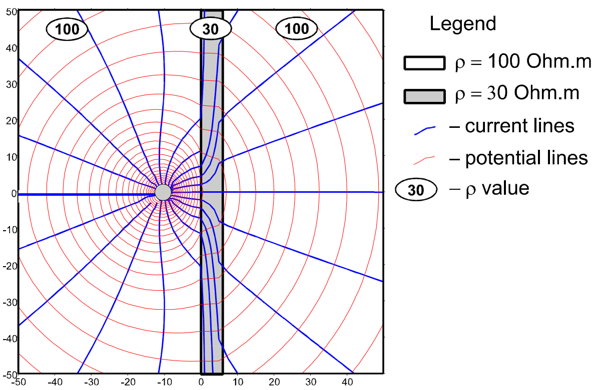
3. Example of ρ_a field's distortions, caused with high resistive object near measuring electrodes. (ρ_{medium}=30, ρ_{body}=100 Ohm.m, XA=100). A - ρ_a -graph, B - current lines distribution in vertical cross-section.



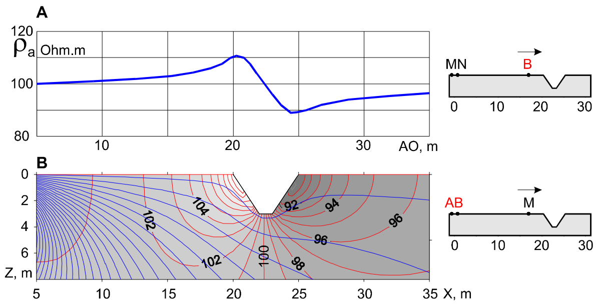
4. Current lines' distribution near high resistive layer with conductive zone in three layered model



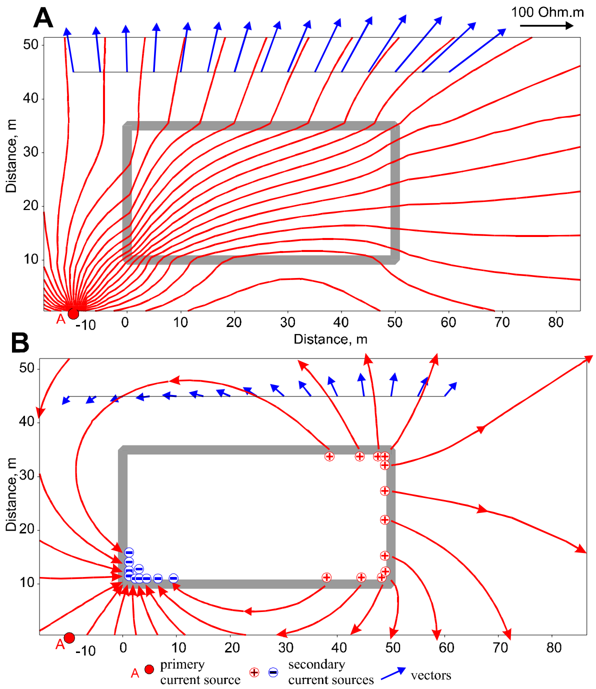
5. A single current electrode near contact with conductive medium



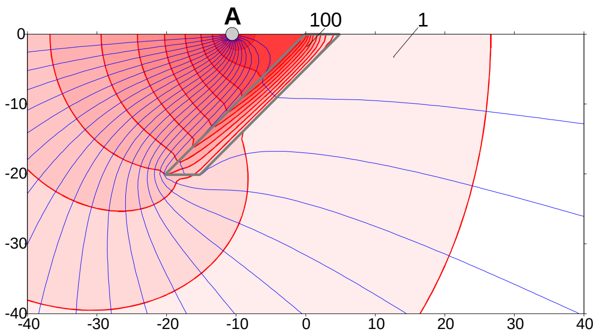
6. A single current electrode near thin conductive layer



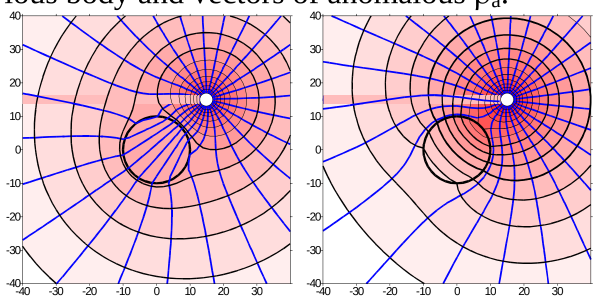
7. ρ_a graph as function r=AO (A), current lines and apparent resistivity distributions (B) for model with negative relief's form. Field technologies are shown on the right.



8. 2D modeling from current source near conductive object (ρ_{Medium}=100, ρ_{body}=15). A. Current lines and ρ_a vectors. B. Current lines of anomalous electric field outside of anomalous body and vectors of anomalous ρ_a.



9. DC current and iso-potential lines near inclined high resistive body



10. DC current and iso-potential lines from point current source near high and low resistive cylinders

For streamlines calculation DC_flow software has been developed. Working version of DC_flow software you can find in Internet at our web-site.

References

Berdichevsky M.N. and Zhdanov M.S. 1984, Advanced Theory Of Deep Geomagnetic Sounding, ELSEVIER, Amsterdam.

Escola, L. 1979, Calculation of galvanic effects by means of the method of sub-areas, Geophysical Prospecting 27, 616-627.

Stratton J.A. 1941, Electromagnetic Theory, McGraw-Hill, New York.